The UNNS Lagrangian Protocol (ULP): Action Principles for Recursive Gauge Fields

UNNS Research Notes

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Abstract

Extending the UNNS Gauge Protocol (UGP), we introduce the *UNNS Lagrangian Protocol* (ULP). This framework defines recursive action principles for UNNS gauge fields, derives Euler–Lagrange recursion equations, and parallels Maxwell and Yang–Mills theories. The ULP establishes UNNS as a variational substrate, opening simulation and quantization pathways.

Contents

1 Motivation

The Gauge Protocol introduced UNNS connections and curvature, but lacked a principle for dynamics. The ULP provides this by defining an *action functional*, from which recursive field equations follow by variation. This aligns UNNS with physical field theory, where Lagrangians govern evolution.

2 Recursive Action Functional

Definition 2.1 (UNNS Action)**.** *Let $\mathcal{F}$ be the recursion curvature. The* UNNS action *is*

$$S[\mathcal{A}] = \int \left(\tfrac{1}{2}\langle \mathcal{F}, \mathcal{F} \rangle + V(\mathcal{A}) \right) d\mu,$$

where $\langle \cdot, \cdot \rangle$ is an inner product on operators, $V(\mathcal{A})$ a potential term, and $d\mu$ a recursive measure.

Remark 2.2. *For abelian UNNS, $V(\mathcal{A}) = 0$ gives a Maxwell-type action. For non-abelian UNNS, interaction terms arise, paralleling Yang–Mills.*

3 Euler–Lagrange Recursion Equations

Theorem 3.1 (Recursive Euler–Lagrange Equations)**.** *Variation of the action with respect to $\mathcal{A}$ yields*

$$D^* \mathcal{F} + \frac{\partial V}{\partial \mathcal{A}} = 0,$$

where D^* is the adjoint recursion derivative.

Proof. Standard variational calculus extended to recursion:

$$\delta S = \int \langle \delta \mathcal{A}, D^* \mathcal{F} + \partial V / \partial \mathcal{A} \rangle d\mu,$$

implying the stated condition. □

4 Examples

4.1 Abelian Case: Recursive Maxwell

$$S = \int \tfrac{1}{2} \mathcal{F}^2 \, d\mu,$$

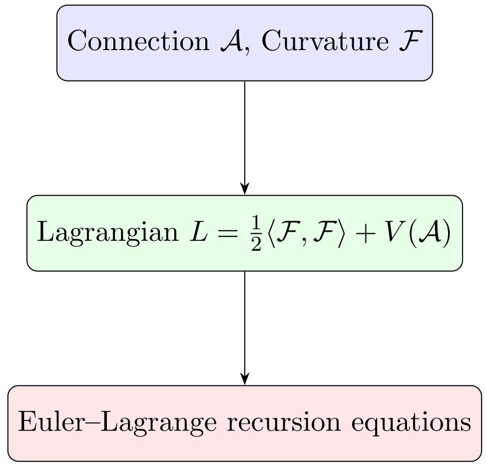
yielding recursion equations analogous to $\nabla \cdot E = 0$.

4.2 Non-Abelian Case: Recursive Yang–Mills

$$S = \int \left(\tfrac{1}{2} \mathcal{F}^2 + \lambda \, \mathrm{Tr}(\mathcal{A}^4) \right) d\mu,$$

yielding non-linear recursion dynamics.

5 Diagrammatic Overview



6 Applications

6.1 Mathematics

- Defines recursive variational calculus.
- Links to discrete action principles in combinatorics.

6.2 Physics

- Provides a recursive analogue of field Lagrangians.
- Suggests pathways to recursive quantum field theories.

6.3 Computation

- Enables UNNS simulation via variational solvers.
- Applications in optimization and machine learning.

7 Conclusion

The UNNS Lagrangian Protocol equips recursion with dynamics via action principles. It aligns UNNS with physics paradigms, bridging discrete recursion with continuous field theories, and opening routes toward quantization.